CASE STUDY: Design of a linkage to guide a point through five specified positions.

(i) Solve the resulting system of equations from the following design problem using Gaussian elimination method. You may also use partial pivoting before the system is solved.

(ii) Solve the same system using one other method and compare the performance of this method in relation to Gaussian elimination method. Discuss the advantages and disadvantages of both methods. You may use MATH-CAD or MATLAB or any other readily available program to do this part.

(iii) Comment also on the condition of this matrix before and after applying partial pivoting.

The five equations in five unknowns appearing below result from the design of a mechanism which is a component of an automatic packaging machine. In this device it is required that a point on the mechanism link be constrained to move over a path having the approximate shape shown in the Figure below. The five points E₁ through E₅ are selected on this curve, with the requirement that the driven point E must pass through each of these five points. A crank length OA is selected, the crank circle is drawn, and for a suitable length EA (arbitrary within limits), the five positions of line EA are determined. A point C is then arbitrarily selected on the link containing line EA. The five positions of point C will in general lie on a conic section since five points determine a conic section. To complete the design of the mechanism, the equation of the conic section passing through the five positions of point C must be determined.

From analytical geometry we recall that the general equation of a conic section is

\[ Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \]


The coefficient A can be considered to have a magnitude of unity, since the entire equation may be divided by the coefficient of \( x^2 \) to establish this value. Appropriate values of the remaining coefficients, B, C, D, E, and F will establish the equation of a conic curve passing through points C₁ through C₅. These coefficient values are determined by solving the five equations formed by substituting the coordinates of each point into the general equation above.
Performing these substitutions, we obtain the following five equations in five unknowns:

\[
\begin{align*}
8.77B + 2.40C + 5.56D + 1.55E + 1.0F &= -32.04 \\
4.93B + 1.21C + 4.48D + 1.10E + 1.0F &= -20.07 \\
3.53B + 1.46C + 2.92D + 1.21E + 1.0F &= -8.53 \\
5.05B + 4.04C + 2.51D + 2.01E + 1.0F &= -6.30 \\
3.54B + 1.04C + 3.47D + 1.02E + 1.0F &= -12.04
\end{align*}
\]

Figure Design of a linkage to guide a point E through five specified positions