The cooling (or heating) of an object is governed by the following heating rate equation:

\[
\frac{dT}{dt} = \frac{A_s h_c}{\rho CV} (T_a - T)
\]

Where \( C \) is the Coefficient of specific heat, \( T \) is the temperature of the object, \( T_a \) is the ambient temperature, \( A_s \) is the surface area and \( V \) is the volume of the object, and \( h_c \) is the convection heat transfer coefficient.

The experimental data given in the table is for a thin plate of ceramic material of size (20cm x 40cm) length and width, respectively. The thickness of the plate is 1cm.

\[
C = 800 \text{ J/kg} \cdot \text{K}; \quad T_a = 400 \text{ K} \quad \rho = 1 \text{ kg/m}^3
\]

Calculate \( \frac{dT}{dt} \) at t=0, t=0.5 and t=1.2

Use these values and the formula given above to calculate the convection heat coefficient \( h_c \), at different times. Also give an average value for \( h_c \). As part of engineering practice give proper units for \( h_c \).

Data Table:

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>T (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>0.1</td>
<td>800</td>
</tr>
<tr>
<td>0.2</td>
<td>700</td>
</tr>
<tr>
<td>0.5</td>
<td>640</td>
</tr>
<tr>
<td>0.8</td>
<td>595</td>
</tr>
<tr>
<td>1.2</td>
<td>470</td>
</tr>
</tbody>
</table>
SOLUTION

First:

Solve for \( \frac{dT}{dt} \) at \( t=0 \) using forward difference

\[
\frac{dT}{dt} = \frac{800 - 1000}{0.1 - 0} = -2000 \text{ } K/s
\]

Solve for \( \frac{dT}{dt} \) at \( t=0.5 \) using central difference

\[
\frac{dT}{dt} = \frac{595 - 700}{0.8 - 0.2} = -175 \text{ } K/s
\]

Solve for \( \frac{dT}{dt} \) at \( t=1.2 \) using backward difference

\[
\frac{dT}{dt} = \frac{470 - 595}{1.2 - 0.8} = -312.5 \text{ } K/s
\]

To solve for \( h_c \), we first need to calculate the volume and surface area of the plate.

\[ V = (0.2m \times 0.4m \times 0.01m) = 0.0008m^3 \]

\[ A = 2 \times (0.2m \times 0.4m) = 0.16m^2 \text{ (x2 to account for both sides of the plate)} \]

Rearranging the given equation, we can solve for \( h_c \)

At \( t=0 \)

\[
h_c = \left( \frac{1 \text{ } kg/m^3}{(0.16m^2)(400K - 1000K)} \right) \left( -2000 \text{ } K/s \right) \left( \frac{800J/\text{kg} \cdot \text{K}}{0.0008m^3} \right) = 13.33 \text{ } W/m^2K
\]

At \( t=0.5 \)

\[
h_c = \left( \frac{1 \text{ } kg/m^3}{(0.16m^2)(400K - 640K)} \right) \left( -175 \text{ } K/s \right) \left( \frac{800J/\text{kg} \cdot \text{K}}{0.0008m^3} \right) = 2.92 \text{ } W/m^2K
\]

At \( t=1.2 \)

\[
h_c = \left( \frac{1 \text{ } kg/m^3}{(0.16m^2)(400K - 470K)} \right) \left( -312.5 \text{ } K/s \right) \left( \frac{800J/\text{kg} \cdot \text{K}}{0.0008m^3} \right) = 17.86 \text{ } W/m^2K
\]

Taking the average of these values

\[
\overline{h_c} = 11.37 \text{ } W/m^2K
\]
Units:

\[
\frac{dT}{dt} = \frac{A_s h_c}{\rho CV} (T_a - T)
\]

Rearranging this equation to solve for \( h_c \)

\[
h_c = \frac{\frac{dT}{dt} \rho CV}{A_s (T_a - T)}
\]

Looking at the units we get

\[
h_c = \frac{(K/s) \left( \frac{kg}{m^3} \right) \left( \frac{J}{kg \cdot K} \right) (m^3)}{(m^2)(K)}
\]

After canceling terms we end up with:

\[
h_c = \frac{J}{s} \cdot m^2 \cdot K
\]

Furthermore, J/s = W, so the final units are \( W/m^2 \cdot K \)